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A Theory of Supersonic Flow past Steady and Oscillating Blunt Bodies of Revolution

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This paper presents a new method of series truncation. The technique is to locate the singularities in the complex plane and then, by using a suitable transformation, to map them away from the region of interest. The method is applied to supersonic flow over both steady and oscillating blunt bodies of revolution. The steady blunt-body solution is obtained by using an inverse method of series truncation with the computation carried out to the third truncation. The steady solution presented yields almost four-figure accuracy throughout the subsonic region, in comparison with known exact solutions. The oscillating blunt-body problem is solved by using a direct method of series truncation with the computation carried out to the second truncation. Two types of motion are considered: "plunging" oscillation and "lunging" oscillation. The oscillation amplitude is assumed to be small; otherwise, within the validity of the governing differential equations, no other restriction is made.

Nomenclature

B	=	bluntness of conic section [see Eq. (1)]
C	=	$1 - B$
C_1,C_2	=	parameters
F,f	=	functions associated with perturbed shock wave
G,g	=	functions associated with perturbed body surface
K,k	=	reduced frequency (referred to V_∞/R_{sk})
M_∞	=	freestream Mach number
p	=	pressure (referred to $\rho_\infty V_\infty^2$)
R_{sk}	=	nose radius of shock wave
t	=	time (referred to R_{sk}/V_∞)
u,v,w	=	velocity components along ξ, η, φ axes
V	=	velocity (referred to freestream value V_∞)
x,r	=	cylindrical polar coordinates
z	=	dependent variable [see Eq. (11)]
α	=	parameter
γ	=	adiabatic exponent
ξ,η,φ	=	curvilinear coordinate system [see Eq. (1)]
ρ	=	density (referred to freestream value ρ_∞)
ϵ	=	amplitude parameter

Superscripts and Subscripts

$(\ )'$	=	ordinary differentiation
ξ,η,φ	=	partial differentiation
b	=	body surface
R,I	=	real part and imaginary part of complex function
$0,0j$	=	functions or parameters associated with steady solution
$1,1j$	=	functions or parameters associated with perturbed solution

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1. Introduction

THE simple blunt-body problem has been extensively investigated. Practically every American aerospace establishment has a machine program for computing blunt-body flows. But, in spite of a wide diversity of methods, there is no real satisfaction with the results because they suffer from marginal accuracy near the sonic line. Van Dyke[1] has pointed out that the difficulty of the various numerical methods arises from the fact that there is a singularity upstream of the flowfield; as a consequence, the radius of convergence no longer includes the entire body surface in the subsonic region. The first objective of this paper is to develop a technique for locating the singularity, and a suitable transformation to map the singularity away from the region of interest. The success of this technique and transformation is demonstrated by obtaining a truncated series that converges over the entire body surface in the subsonic region.

The second objective is to study flowfield disturbances in the subsonic region of supersonic flows over oscillating blunt bodies of revolution. The problem is of considerable interest in connection with manned and unmanned space vehicle design, particularly with regard to the effect of flowfield disturbances emanating from unsteady periodic motion due to gust loading, elastic deformation, structural vibration, engine-induced vibration, and their interaction with the automatic guidance and stability system. Such information is useful for estimating local stress distribution, panel loading, trajectory deviation, dynamic stability, and the like.

Hsu and Ashley[2] have suggested a number of computational methods for obtaining unsteady surface pressure on blunt-nosed missiles, but have presented no results. Kennet[3] has used a constant-density approximation to compute aerodynamic loads on a blunt-nosed missile performing small-amplitude harmonic oscillation in a hypersonic stream. In that analysis he has assumed that the detached shock wave is concentric with the body surface, that the lateral velocity in the steady solution is linear along the body surface, and that the flow is quasi-steady. Telenin and Lipnitskii[4] have applied small-perturbation theory to determine the nonstationary and quasi-steady supersonic flow around a blunt body performing a small angular oscillation (pitching). Sauerwein[5,6] has developed a general program that employs the method of characteristics to compute the flowfield of arbitrary multidimensional and unsteady flow; however, he has given no specific applications with his numerical results.

In this paper the oscillation amplitude is assumed to be small; otherwise, within the validity of the governing differential equations, no other restrictions are made. The amplitude restriction can be removed if the analysis is extended to higher orders. The extension, although lengthy, is straightforward.

2. Formulation of the Problem

The coordinate system adopted here is that developed by Van Dyke.[7] Let a detached shock wave be described by a conic section. Then, in cylindrical polar coordinates originating from its vertex (Fig. 1), any such shock wave may be described by

$$r^2 = 2R_{sk}x - Bx^2$$

where B is the bluntness parameter. The bluntness is zero for a paraboloid, negative for hyperboloids, positive for ellipsoids, and unity for a sphere. An orthogonal coordinate system (ξ,η), containing the shock wave as one of the coordinate surfaces, is introduced by setting

$$x/R_{sk} = (1/B)[1 - (1 - B\xi^2)^{1/2}(C + B\eta^2)^{1/2}]$$
$$r/R_{sk} = \xi\eta \tag{1a}$$

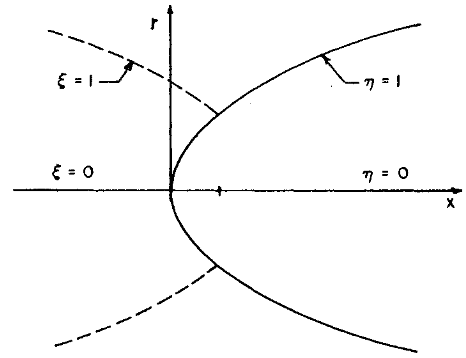


Fig. 1 Conic-section coordinate system.

where $C = 1 - B$. A special case is the paraboloid with

$$x/R_{sk} = \tfrac{1}{2}(1 + \xi^2 - \eta^2) \quad \text{for } B = 0 \tag{1b}$$

The shock wave is described by $\eta = 1$. Let the azimuthal angle, φ, be the third orthogonal coordinate; then the differential line element, ds, is given by

$$\left(\frac{ds}{R_{sk}}\right)^2 = \frac{C\xi^2 + \eta^2}{1 - B\xi^2}(d\xi)^2 + \frac{C\xi^2 + \eta^2}{C + B\eta^2}(d\eta)^2 + (\xi\eta)^2(d\varphi)^2$$

2.1 Equation for Rigid Body Motion

Let the body oscillate harmonically with small amplitude in a space-fixed Newtonian frame with respect to which the freestream has a uniform velocity V_∞. The equations describing the plunging motion of the body and of the shock wave to the first order in the amplitude of oscillation are respectively given by (Fig. 2)

$$G(\xi,\eta,t) = \eta - \{\eta_0(\xi) + \mathcal{R}\mathrm{e}[g(\xi)\cos\varphi\epsilon e^{ikt} + \ldots]\} = 0 \tag{2}$$

and

$$F(\xi,\eta,t) = \eta - \{1 + \mathcal{R}\mathrm{e}[f(\xi)\cos\varphi\epsilon e^{ikt} + \ldots]\} = 0 \tag{3}$$

where

$$g(\xi) = \xi(C + B\eta_0^2)/[C\xi^2 + \eta_0^2]$$

Here $\eta_0(\xi)$ is an even function of ξ which describes the shape of an axisymmetric body, $f(\xi)$ is an unknown function, and $\mathcal{R}\mathrm{e}$ indicates the real part only.

2.2 Differential Equations

Let all lengths be referred to the nose radius of the shock wave R_{sk}, velocities to the freestream speed V_∞; density to the freestream value ρ_∞; and pressure to $\rho_\infty V_\infty^2$. Then the differential equations in the (ξ,η,φ) coordinate system may be written as follows:

$$\frac{\partial\rho}{\partial t} + \frac{[(C + B\eta^2)(1 - B\xi^2)]^{1/2}}{\xi\eta(C\xi^2 + \eta^2)}\Bigg\{\left[\xi\eta\left(\frac{C\xi^2 + \eta^2}{C + B\eta^2}\right)^{1/2}\rho u\right]_\xi + \left[\xi\eta\left(\frac{C\xi^2 + \eta^2}{1 - B\xi^2}\right)^{1/2}\rho v\right]_\eta + \left[\frac{C\xi^2 + \eta^2}{(1 - B\xi^2)^{1/2}(C + B\eta^2)^{1/2}}\rho w\right]_\varphi\Bigg\} = 0 \tag{4a}$$

$$\frac{\partial u}{\partial t} + \left(\frac{1 - B\xi^2}{C\xi^2 + \eta^2}\right)^{1/2}\Bigg[uu_\xi - \frac{C\xi}{C\xi^2 + \eta^2}v^2 + \left(\frac{C + B\eta^2}{1 - B\xi^2}\right)^{1/2} v\left(u_\eta + \frac{\eta}{C\xi^2 + \eta^2}u\right) + \left(\frac{C\xi^2 + \eta^2}{1 - B\xi^2}\right)^{1/2}\frac{w}{\xi\eta}u_\varphi - \frac{w^2}{\xi} + \frac{1}{\rho}p_\xi\Bigg] = 0 \tag{4b}$$

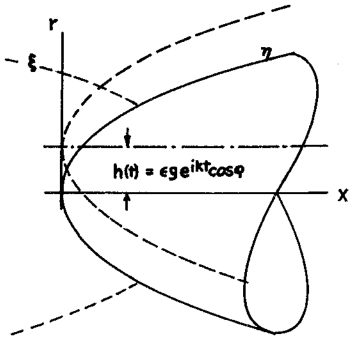


Fig. 2 Plunging displacement.

$$\frac{\partial v}{\partial t} + \left(\frac{C + B\eta^2}{C\xi^2 + \eta^2}\right)^{1/2}\left[v v_\eta - \frac{\eta}{C\xi^2 + \eta^2}u^2 + \left(\frac{1 - B\xi^2}{C + B\eta^2}\right)^{1/2} \times u\left(v_\xi + \frac{C\xi}{C\xi^2 + \eta^2}v\right) + \left(\frac{C\xi^2 + \eta^2}{C + B\eta^2}\right)^{1/2}\frac{w}{\xi\eta}v_\varphi - \frac{w^2}{\eta} + \frac{1}{\rho}p_\eta\right] = 0 \quad (4c)$$

$$\frac{\partial w}{\partial t} + \left(\frac{1 - B\xi^2}{C\xi^2 + \eta^2}\right)^{1/2} u\left[w_\xi + \frac{w}{\xi}\right] + \left(\frac{C + B\eta^2}{C\xi^2 + \eta^2}\right)^{1/2} \times v\left[w_\eta + \frac{w}{\eta}\right] + \frac{w}{\xi\eta}w_\varphi + \frac{1}{\xi\eta\rho}p_\varphi = 0 \quad (4d)$$

$$\frac{\partial}{\partial t}(p/\rho^\gamma) + u\left(\frac{1 - B\xi^2}{C\xi^2 + \eta^2}\right)^{1/2}(p/\rho^\gamma)_\xi + v\left(\frac{C + B\eta^2}{C\xi^2 + \eta^2}\right)^{1/2}(p/\rho^\gamma)_\eta + \frac{w}{\xi\eta}(p/\rho^\gamma)_\varphi = 0 \quad (4e)$$

These equations represent the conservation of mass, momentum, and entropy of a perfect gas.

In accordance with Eqs. (2) and (3), the dependent variables in Eqs. (4) may be written as follows:

$$p(\xi,\eta,t) = p_0(\xi,\eta) + \mathrm{Re}[p_1(\xi,\eta)\cos\varphi\epsilon e^{ikt}] + \ldots \quad (5a)$$

$$\rho(\xi,\eta,t) = \rho_0(\xi,\eta) + \mathrm{Re}[\rho_1(\xi,\eta)\cos\varphi\epsilon e^{ikt}] + \ldots \quad (5b)$$

$$u(\xi,\eta,t) = u_0(\xi,\eta) + \mathrm{Re}[u_1(\xi,\eta)\cos\varphi\epsilon e^{ikt}] + \ldots \quad (5c)$$

$$v(\xi,\eta,t) = v_0(\xi,\eta) + \mathrm{Re}[v_1(\xi,\eta)\cos\varphi\epsilon e^{ikt}] + \ldots \quad (5d)$$

$$w(\xi,\eta,t) = \mathrm{Re}[w_1(\xi,\eta)\sin\varphi\epsilon e^{ikt}] + \ldots \quad (5e)$$

where $(\ )_0$ stands for the steady-state solution and $(\ )_1$ represents the unsteady perturbed and complex solution. By substituting Eqs. (5) into Eqs. (4) and equating the coefficients for like powers of ϵ to zero, the leading terms are the differential equations for the steady-state flow and the second terms are the set of linear differential equations for the first-order unsteady perturbation.

2.3 Boundary Conditions at the Shock Wave and on the Body Surface

From the oblique shock wave relations the following boundary conditions are obtained at the mean position of the shock wave:

1) for the steady-state differential equations

$$p_0(\xi,1) = [2/(\gamma + 1)][(1 - B\xi^2)/(1 + C\xi^2)] - [(\gamma - 1)/(\gamma + 1)][1/(\gamma M_\infty^2)] \quad (6a)$$

$$1/\rho_0(\xi,1) = [(\gamma - 1)/(\gamma + 1)] + [2/(\gamma + 1)] \times [(1 + C\xi^2)/(1 - B\xi^2)](1/M_\infty^2) \quad (6b)$$

$$u_0(\xi,1) = \xi/(1 + C\xi^2)^{1/2} \quad (6c)$$

$$v_0(\xi,1) = -\left(\frac{1 - B\xi^2}{1 + C\xi^2}\right)^{1/2}\left(\frac{\gamma - 1}{\gamma + 1} + \frac{2}{\gamma + 1}\frac{1 + C\xi^2}{1 - B\xi^2}\frac{1}{M_\infty^2}\right) \quad (6d)$$

2) for the first-order unsteady differential equations

$$p_1(\xi,1) = \frac{4}{\gamma + 1}\frac{1 - B\xi^2}{1 + C\xi^2}\left[ikf\frac{1 + C\xi^2}{(1 - B\xi^2)^{1/2}} + \xi\frac{df}{d\xi} + \frac{C\xi^2}{1 + C\xi^2}f\right] - f\frac{\partial}{\partial\eta}p_0\bigg|_{\eta=1} \quad (7a)$$

$$\rho_1(\xi,1) = \frac{4}{\gamma + 1}\frac{1 + C\xi^2}{1 - B\xi^2}\frac{\rho_0^2}{M_\infty^2}\left[ikf\frac{1 + C\xi^2}{(1 - B\xi^2)^{1/2}} + \xi\frac{df}{d\xi} + \frac{C\xi^2}{1 + C\xi^2}f\right] - f\frac{\partial}{\partial\eta}\rho_0\bigg|_{\eta=1} \quad (7b)$$

$$u_1(\xi,1) = \frac{1 - B\xi^2}{(1 + C\xi^2)^{1/2}}\left[\frac{2}{\gamma + 1}\left(-1 + \frac{1}{M_\infty^2}\frac{1 + C\xi^2}{1 - B\xi^2}\right)\frac{df}{d\xi} - \frac{C\xi}{1 + C\xi^2}f\right] - f\frac{\partial}{\partial\eta}u\bigg|_{\eta=1} \quad (7c)$$

$$v_1(\xi,1) = \left(\frac{1 - B\xi^2}{1 + C\xi^2}\right)^{1/2}\left\{\left(\frac{2}{\gamma + 1}\frac{1}{M_\infty^2}\frac{2 - C\xi^2}{1 - B\xi^2} - \frac{\gamma - 1}{\gamma + 1}\frac{1}{1 + C\xi^2}C\xi^2\right)f + \frac{2}{\gamma + 1}\left(1 + \frac{1}{M_\infty^2}\frac{1 + C\xi^2}{1 - B\xi^2}\right) \times \left[\xi\frac{df}{d\xi} + ikf\frac{1 + C\xi^2}{(1 - B\xi^2)^{1/2}}\right]\right\} - f\frac{\partial}{\partial\eta}v_0\bigg|_{\eta=1} \quad (7d)$$

$$w_1(\xi,1) = \frac{(1 - B\xi^2)^{1/2}}{\xi}\frac{2}{\gamma + 1}\left(1 - \frac{1}{M_\infty^2}\frac{1 + C\xi^2}{1 - B\xi^2}\right)\cdot f \quad (7e)$$

The boundary condition on the body surface is given by

$$\partial G/\partial t + \mathbf{V}\cdot\nabla G = 0 \quad \text{at } G = 0 \quad (8)$$

First, using the present coordinate system, then transferring all variables from the body surface to the mean position of the body surface and equating like powers of ϵ to zero, one obtains

$$\left(\frac{C + B\eta^2}{C\xi^2 + \eta_0^2}\right)^{1/2} v_0 - \left(\frac{1 - B\xi^2}{C\xi^2 + \eta_0^2}\right)^{1/2} u_0\eta_{0\xi} = 0 \quad \text{at } \eta = \eta_0(\xi) \quad (9)$$

for the steady-state differential equations and

$$-ikg + \left(\frac{C + B\eta_0^2}{C\xi^2 + \eta_0^2}\right)^{1/2}\left[v_1 + v_{0\eta}g - \frac{C\eta_0 g(1 - B\xi^2)}{(C\xi^2 + \eta_0^2)(C + B\eta_0^2)}v_0\right] - \left(\frac{1 - B\xi^2}{C\xi^2 + \eta_0^2}\right)^{1/2}\left[u_0\left(g_\xi - \frac{g\eta_0}{C\xi^2 + \eta_0^2}\eta_{0\xi}\right) + (u_1 + u_{0\eta}g)\eta_{0\xi}\right] = 0 \quad \text{at } \eta = \eta_0(\xi) \quad (10)$$

for the first-order perturbed differential equations.

3. Method of Solution

3.1 Steady-State Solution

Van Dyke[1,7] has pointed out that in the inverse blunt-body problem a singular limiting line appears in the analytical continuation of the series solution upstream of the bow shock wave. He has suggested that convergence of the series may be improved simply by choosing the independent variable so that the singularities are removed far away from the domain of interest. For example, Swigart[8] uses a power series expansion in ξ^2 for all dependent variables to solve an inviscid supersonic flow around a blunt body at nonzero angle of attack. The resulting surface pressure distribution near the sonic line oscillates in successive truncation. Based on physical insight, Van Dyke[9] postulates two singularities at $\xi = \pm i$ in the complex plane and then introduces an expansion in powers of $\xi^2/(1 + \xi^2)$ instead of ξ^2. The resulting surface pressure in the second approximation is correct to almost four significant figures throughout the subsonic region.

Usually, when a series expansion is used for a set of nonlinear differential equations one knows neither the nature nor the location of the singularities in the complex plane which limit the convergence of the series. Van Dyke's approach can, then, be extended by introducing a parameter α and by recasting the series in powers of a new variable

$$z^2 = \xi^2/[1 + \alpha(1 - \eta)\xi^2] \tag{11}$$

The roots in the denominator of the above expression may be considered as the singularities in the complex plane. The unknown parameter α is to be determined along with a numerical solution of the differential equations in such a way that an improved convergence of the series solution is obtained within the region of interest.

In accordance with the initial conditions at the shock wave the solutions are expanded as follows:

$$p_0(\xi,\eta) = [1/(C\xi^2 + \eta^2)][p_{01}(\eta) + p_{02}(\eta)z^2 + p_{03}(\eta)z^4 + \dots] \tag{12a}$$

$$\rho_0(\xi,\eta) = (1 - B\xi^2)[\rho_{01}(\eta) + \rho_{02}(\eta)z^2 + \rho_{03}(\eta)z^4 + \dots] \tag{12b}$$

$$u_0(\xi,\eta) = \frac{(C + B\eta^2)^{1/2}}{\xi\eta(C\xi^2 + \eta^2)^{1/2}}[u_{01}(\eta)z^2 + u_{02}(\eta)z^4 + u_{03}(\eta)z^6 + \dots] \tag{12c}$$

$$v_0(\xi,\eta) = \{1/[\xi\eta(1 - B\xi^2)^{1/2}(C\xi^2 + \eta^2)^{1/2}]\} \times [v_{01}(\eta)z + v_{02}(\eta)z^3 + v_{03}(\eta)z^5 + \dots] \tag{12d}$$

The above expansion simplifies not only the initial conditions at the shock wave but also the governing differential equations and is selected after a systematic trial of several alternatives.

If Eqs. (12) are substituted into Eqs. (4) (with $\partial/\partial t$ terms omitted and $w = 0$) and the coefficients of like powers of z are equated to zero, one obtains a set of nonlinear ordinary differential equations in the following form:

$$\begin{aligned} p_{0j}' &= f_1(\eta;\, p_{01},\rho_{01},u_{01},v_{01}, \dots p_{0j},\rho_{0j},u_{0j},v_{0j}) \\ \rho_{0j}' &= f_2(\eta;\, p_{01},\rho_{01},u_{01},v_{01}, \dots p_{0j},\rho_{0j},u_{0j},v_{0j}) \\ u_{0j}' &= f_3(\eta;\, p_{01},\rho_{01},u_{01},v_{01}, \dots p_{0j},\rho_{0j},u_{0j},v_{0j}) + g_3(\eta;\, p_{0j+1}) \\ v_{0j}' &= f_4(\eta;\, p_{01},\rho_{01},u_{01},v_{01}, \dots p_{0j},\rho_{0j},u_{0j},v_{0j}) \end{aligned} \tag{13}$$

for $j = 1,2,3$. Clearly it is not possible to solve these equations successively because the number of unknowns always exceeds the number of the equations by one. This extra unknown arises from the elliptic nature of the differential equations and can be interpreted physically as the backward influence within the subsonic region. Fortunately this extra unknown is the surface pressure p_{0j+1} which appears only in the differential equation for u. When the velocity component u is expanded in power series of z the error introduced by neglecting this higher order term, p_{0j+1}, is small within the subsonic region where $z < 1$. Furthermore, the velocity component u vanishes along the stagnation streamline and has a small magnitude at the sonic point. Thus, the extra unknown has very little influence on the solution and one may set it equal to zero. The resulting differential equations may then be solved in succession. Note that, at the shock wave $z = \xi$, the required initial conditions for Eqs. (13) are obtained by equating the coefficients of like powers of z between Eqs. (12) and (6).

Because of axial symmetry, the equation for the body surface may be written

$$\eta_0(z) = \eta_{01} + \eta_{02}z^2 + \eta_{03}z^4 + \dots \tag{14}$$

where $z^2 = \xi^2/[1 + (1 - \eta_{01})\xi^2]$, and η_{0j} are constants and are determined from the boundary condition on the body surface. Once the shape of the body surface is known, the steady-state solution on the body surface is readily obtained by using a Taylor series expansion from the $\eta = \eta_{01}$ line to the actual body surface. For example, the equation for the surface pressure is given by

$$p_0 = [1/(C\xi^2 + \eta_0^2)][\bar{p}_{01} + \bar{p}_{02}z^2 + \bar{p}_{03}z^4 + \dots] \quad \text{at } \eta = \eta_0(z) \tag{15}$$

where

$$\bar{p}_{01} = p_{01}, \quad \bar{p}_{02} = p_{02} + \eta_{02}p_{01}'$$

$$\bar{p}_{03} = p_{03} + \eta_{02}p_{02}' + \eta_{03}p_{01}' + \tfrac{1}{2}\eta_{02}^2 p_{01}''$$

Expressions for other variables may be written in a similar manner.

3.2 Plunging Oscillation

Expanding Eq. (2) in power series in z results in

$$\eta = \eta_0(z) + \Re e[(g_1 z + g_2 z^3 + \dots)\cos\varphi\epsilon e^{ikt}] \tag{16}$$

where

$$g_1 = (C + B\eta_{01}^2)/\eta_{01}^2$$

$$g_2 = g_1[\tfrac{1}{2}\alpha(1 - \eta_{01}) - (C/\eta_{01}^2)] - (2C\eta_{02}/\eta_{01}^3)$$

This is the equation for the prescribed plunging motion of a rigid body (Fig. 2). It follows that the equation for the shock wave [Eq. (3)] may be written

$$\eta(z) = 1 + \Re e f(z)\cos\varphi\epsilon e^{ikt} + \dots \tag{17}$$

where

$$f(z) = f_1 z + f_2 z^3 + \dots$$

Substituting Eqs. (17) into Eqs. (7) and equating the coefficients of like powers of z to zero provides the required boundary conditions at the shock wave for the unsteady perturbed differential equations. In accordance with Eqs. (12) and (17) and the boundary conditions at the shock wave, the perturbed solutions may be expanded as follows:

$$p_1(\xi,\eta) = [1/(C\xi^2 + \eta^2)^2][p_{11}(\eta)z + p_{12}(\eta)z^3 + \dots] \tag{18a}$$

$$\rho_1(\xi,\eta) = (1 - B\xi^2)[\rho_{11}(\eta)z + \rho_{12}(\eta)z^3 + \dots] \tag{18b}$$

$$u_1(\xi,\eta) = \frac{(C + B\eta^2)^{1/2}}{\xi\eta(C\xi^2 + \eta^2)^{1/2}}[u_{11}(\eta)z + u_{12}(\eta)z^3 + \dots] \tag{18c}$$

$$v_1(\xi,\eta) = \{1/[\xi\eta(C\xi^2 + \eta^2)^{1/2}(1 - B\xi^2)^{1/2}]\} \times [v_{11}(\eta)z^2 + v_{12}(\eta)z^4 + \dots] \tag{18d}$$

$$w_1(\xi,\eta) = \frac{(C + B\eta^2)^{1/2}}{(C\xi^2 + \eta^2)(1 - B\xi^2)^{1/2}}[w_{11}(\eta) + w_{12}(\eta)z^2 + \dots] \tag{18e}$$

Substituting the above expressions into the unsteady perturbed differential equations and equating like powers of z to zero results in a system of linear ordinary differential equations in the following form:

$$p_{1n}' = f_1(\eta;\, p_{0n},\rho_{0n},u_{0n},v_{0n}, \dots, p_{1n},\rho_{1n},u_{1n},v_{1n},w_{1n}) + g_1(\eta;\, u_{1n+1},w_{1n+1}) \tag{19a}$$

$$\rho_{1n}' = f_2(\eta;\, p_{0n},\rho_{0n},u_{0n},v_{0n}, \dots, p_{1n},\rho_{1n},u_{1n},v_{1n},w_{1n}) + g_2(\eta;\, u_{1n+1},w_{1n+1}) \tag{19b}$$

$$u_{1n}' = f_3(\eta;\, p_{0n},\rho_{0n},u_{0n},v_{0n},\, \ldots,\, p_{1n},\rho_{1n},u_{1n},v_{1n},w_{1n}) \quad (19c)$$

$$v_{1n}' = f_4(\eta;\, p_{0n},\rho_{0n},u_{0n},v_{0n},\, \ldots,\, p_{1n},\rho_{1n},u_{1n},v_{1n},w_{1n}) + g_4(\eta;\, u_{1n+1},w_{1n+1}) \quad (19d)$$

$$w_{1n}' = f_5(\eta;\, p_{0n},\rho_{0n},u_{0n},v_{0n},\, \ldots,\, p_{1n},\rho_{1n},u_{1n},v_{1n},w_{1n}) \quad \text{for } n = 1,2\ldots \quad (19e)$$

Here the number of unknowns always exceeds the number of the differential equations by two, i.e., u_{1n+1} and w_{1n+1}. In the nth truncation,

$$u_{1n+1}'(\eta) = 0, \quad w_{1n+1}'(\eta) = 0$$

It follows that u_{1n+1} and w_{1n+1} can be either zeroes or constants depending upon their influence to the perturbed solutions. In the present case, it is evident that they exert a strong influence and hence should be assigned some nonzero constants. The method of finding them is given below.

First, if the shock parameters f_n are known, then u_{1n+1} and w_{1n+1} just behind the shock wave are known also. Second, in the limit of zero frequency the motion is physically equivalent to a coordinate translation, hence, the perturbed solutions should approach the steady-state solutions at the displaced position, $\eta = \eta_0(z) + g(z)\cos\varphi\epsilon$. These two conditions are sufficient to provide general criteria to estimate what values may be assigned for u_{1n+1} and w_{1n+1} in Eqs. (19).

Consider for the case $n = 2$. The boundary conditions at the shock wave show that $f_n = 0$ for $n \geq 3$. The differential equations state that u_{13} and w_{13} are independent of η and, hence, are independent of f_n [Eq. (17)]. In evaluating u_{13} and w_{13} just behind the shock wave one need not consider f_n for $n \geq 3$. Let the shock parameters f_1 and f_2 be multiplied by two constants C_1 and C_2, respectively. One obtains u_{13} and w_{13} just behind the shock wave

$$u_{13} = \left[\frac{6}{\gamma+1}\left(B + \frac{C}{M_\infty^2}\right) - u_{01}' + 1\right]\cdot(C_2f_2) - [u_{02}' + \alpha]\cdot(C_1f_1) + \ldots \quad (20a)$$

$$w_{13} = \frac{2}{\gamma+1}\left[\left(2C - 1 - \frac{2C}{M_\infty^2}\right)\cdot(C_2f_2) - \left(B + \frac{C}{M_\infty^2}\right) C\cdot(C_1f_1)\right] + \ldots \quad (20b)$$

If u_{13} and w_{13} in Eqs. (19) be set equal to those given in the preceding equations with $C_1 = C_2 = 1$, one finds that some of the perturbed solutions on the body surface at zero frequency exceed the steady-state solutions in the displaced position by a maximum of about 8%. With $C_1 = C_2 = 0$, the perturbed solutions are too small by about 12%. This suggests that the perturbed solutions may be improved if C_1 and C_2 are assigned some values between zero and one. Initial guesses are adjusted in accordance with numerical solutions so that at zero frequency one of the perturbed solutions, say surface pressure, approaches the steady-state value in a displaced position. At $k = 0$, the absolute magnitude of p_1 or p_{Rb} ($p_1 = p_{Rb} + ip_{Ib}$ and $p_{Ib} = 0$ exactly) is less than 0.01 throughout the subsonic region (Fig. 9) and the constants are found to be

$$\begin{aligned} C_1 &= 1.0 \quad C_2 = 0.8 \quad \text{for } B = 0, M_\infty = 10^4 \\ C_1 &= 0.51 \quad C_2 = 1.0 \quad \text{for } B = 1, M_\infty = 10^4 \end{aligned} \quad (21)$$

Equivalently

$$\begin{aligned} u_{13} &= -4.12f_1 + 4.70f_2 \quad w_{13} = 0.666f_2 \quad \text{for } B = 0 \\ u_{13} &= -2.10f_1 + 8.35f_2 \quad w_{13} = -0.833f_2 \quad \text{for } B = 1 \end{aligned}$$

In finding C_1 and C_2, one of them is more sensitively dependent on the perturbed solution than the other. Let the less sensitive one be set equal to unity, then the other can be easily determined. Even when the values for C_1 and C_2 are given only one significant figure, the resulting solutions at zero frequency are greatly improved. By careful adjustment much more accurate solutions are, of course, possible. When the surface pressure improves, the other solutions, such as the parameters, f_n, at the shock wave and other variables, ρ,u,v, on the body surface improve in proportion. Hence, it does not matter which perturbed solution is used for assigning the C_1 and C_2. The results are the same. Furthermore, for $0 < (C_1,C_2) < 1$ there exists only one set of values such that the above mentioned two conditions are both satisfied. Thus, the solutions are uniquely determined.

Note that u_{13} and w_{13} in Eqs. (19) are slightly less than these just behind the shock wave. This may indicate the nature of rapid convergence of the series solution.

From the equation for the prescribed motion of a rigid body, it is evident that the series truncation error depends mainly upon the body bluntness parameter and is independent of the frequency. In addition, the average value for the velocity components, u_1w_1, just behind the shock wave is slightly larger than that on the body surface at zero and low frequency. This is also true at high frequency. The constants C_1 and C_2 which are used to compensate for the deviation of u_1 and w_1 in the higher approximation within the shock layer can, therefore, be expected of the same order of magnitude over the entire frequency range. Thus, the constants determined for the zero frequency will also be valid for all nonzero frequencies.

The boundary conditions on the body surface for the perturbed differential equations may be obtained as follows. First, the velocity components [Eqs. (18)] are substituted into the boundary condition for the perturbed differential equations [Eq. (10)]. Second, all variables are transformed from the body surface to the constant η line passing through the stagnation point. Third, the coefficients for like powers of z are set equal to zero, that gives a set of algebraic equations which determines the f_n.

By reverting the transformation, all solutions are transformed from the mean position to the body surface. The perturbed solutions are then obtained. For example, the resultant surface pressure,

$$p_b = p_0 + \Re\mathrm{e}\left\{\frac{\partial p_0}{\partial\eta}\Delta\eta + \frac{\partial p_0}{\partial\xi}\Delta\xi + \bar{p}_1\cos\varphi\epsilon e^{ikt} + \ldots\right\}_{\eta=\eta_0} \quad (22)$$

where

$$\Delta\eta = (g_1z + g_2z^3 + \ldots)\cos\varphi\epsilon e^{ikt}$$

$$\Delta\xi = \frac{1 - B\xi^2}{C + B\eta^2}\frac{\eta}{\xi}\Delta\eta = \frac{\eta_{01}}{C + B\eta_{01}^2}\left[1 + \left(\frac{\eta_{02} - B\eta_{01}}{\eta_{01}} - \frac{2B\eta_{01}\eta_{02}}{C + B\eta_{01}^2}\right)z^2 + \ldots\right]\frac{\Delta\eta}{\xi}$$

$$\bar{p}_1 = [1/(C\xi^2 + \eta_0^2)^2][p_{11}z + (p_{12} + \eta_{02}p_{11}')z^3 + \ldots]$$

3.3 Lunging Oscillation

The equations for the lunging motion of the rigid body and of the shock wave are, respectively, given by Fig. 3

$$G(\xi,\eta,t) = \eta - [\eta_0 + \Re\mathrm{e}\,g(\xi)\epsilon e^{ikt} + \ldots] = 0 \quad (23)$$

and

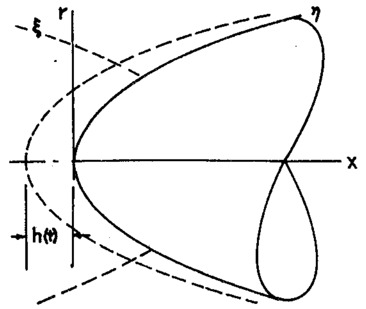
$$F(\xi,\eta,t) = \eta - [1 + \Re\mathrm{e}(f_1 + f_2\xi^2 + f_3\xi^4 + \ldots)\epsilon e^{ikt}] = 0 \quad (24)$$

where

$$g(\xi) = \eta_0(1 - B\xi^2)^{1/2}(C + B\eta_0^2)^{1/2}/(C\xi^2 + \eta_0^2)$$

Following the steps previously outlined for the case of plunging, one finds in the second truncation that one of the

Fig. 3 Lunging displacement.



two extra unknowns, f_3, appears in the boundary conditions at the shock wave and the other, p_{13}, in the differential equation for u. It is well known that for a blunt-body problem a minute local change of the shock wave shape affects the solution significantly.[11,12] Here also the extra unknown f_3 is much more important than p_3; therefore, the latter may be set equal to zero and the former is determined as follows.

The shock wave function at the stationary displaced position is obtained by setting $t = 0$ and $\eta_0 = 1$ in Eq. (23)

$$\eta = 1 + [1 - (C + \tfrac{1}{2}B)\xi^2 + (C^2 + \tfrac{1}{2}BC - B^2/8)\xi^4 + \ldots]\epsilon \quad (25)$$

Equating the coefficients of ξ from Eqs. (24) and (25) one may express f_3 in terms of f_1 and f_2

$$f_3 = (C^2 + \tfrac{1}{2}BC - B^2/8)\cdot(C_1f_1) - [(C^2 + \tfrac{1}{2}BC - B^2/8)/(C + \tfrac{1}{2}B)](C_2f_2) \quad (26)$$

To remedy the variation in the unsteady oscillation two unknown constants C_1 and C_2 have again been introduced. The method of determining these two constants is similar to that outlined for the case of plunging. That is, at $k = 0$, the perturbed surface pressure approaches the steady-state solution in a displaced position.

4. Results and Discussion

4.1 Steady-State Solution

The steady-state solution is obtained by using an inverse method of a given shock. The computation is carried out to the third truncation. The parameter α is determined by matching the surface pressures in the second and third truncations in the neighborhood of the sonic point, ($\xi = 0.7$) until they agree to four significant figures. It is found that if these surface pressures agree to four figures at the sonic point, then they agree to far more than four figures towards the stagnation region. It is also found that by carefully adjusting the parameter α a still higher accuracy can be obtained. Figure 4 shows a typical effect of α on the surface pressure distribution in the second and third truncations.

When the differential equations are solved, α is initially unknown. Therefore, each solution requires three or four iterations. For a UNIVAC 1108 computer this usually requires less than half a minute machine time. The rate of convergence for the surface pressure in the first four truncations is so rapid that inclusion of the fourth truncation does not affect even the fourth place of the surface pressure results. However, the calculation of the fourth truncation requires a double-precision integration routine.

Perry[10] has summarized various analytic solutions of the flowfield in the nose region of a steady-state blunt body at supersonic speeds. Figures 5 and 6 are typical examples for the surface pressure distributions. The methods of solution used by these agencies differ from each other. For example, AVCO, Massachusetts Institute of Technology (MIT), Douglas 2, and Naval Weapons Laboratory (NWL) use the one-strip Belotserkovskii method. General Applied Science Laboratory (GASL) 3 is the time-dependent method of Moretti. Forward-marching finite-difference methods are used by NASA, Northrop, and Douglas 1. General Electric (GE) uses the method of Gravalos. GASL 1 employs the method of Vaglio-Laurin and Ferri. GASL 2 and Lockheed

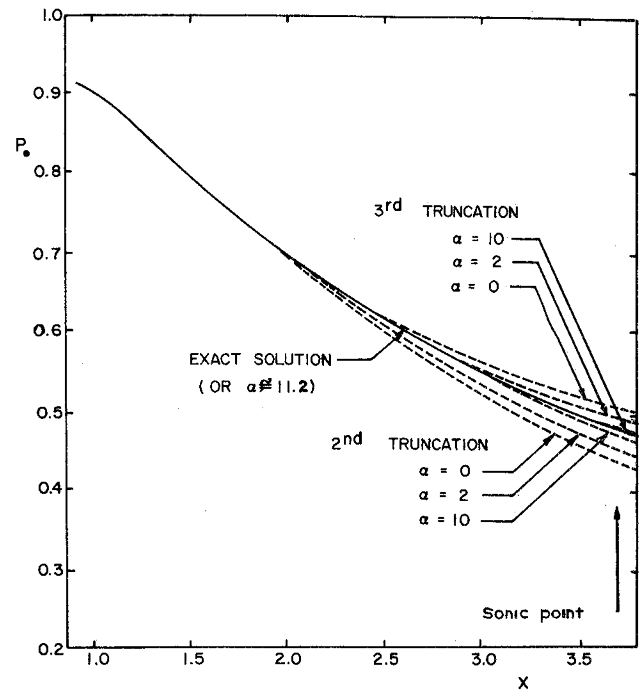


Fig. 4 Variation of steady-state surface pressure with α ($M_\infty = 10^4$, $B = 0$, and $\gamma = 1.4$).

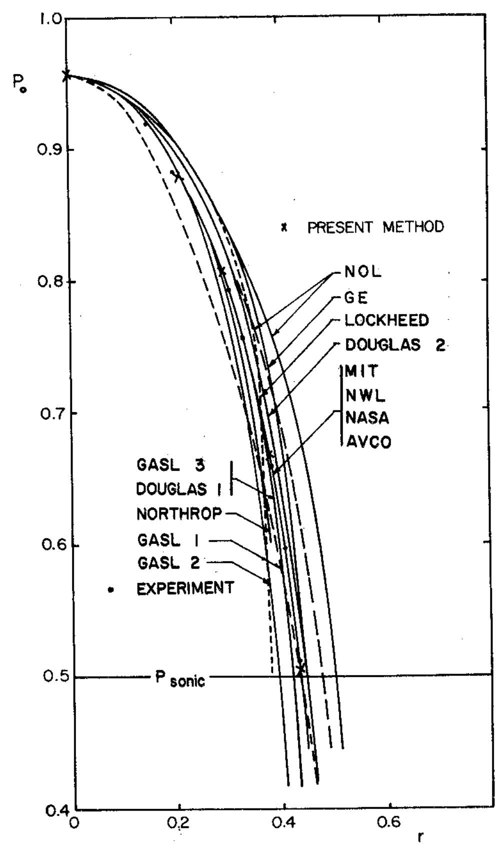


Fig. 5 Steady-state surface pressure ($B_b = 4$, $M_\infty = 3$, and $\gamma = 1.4$).

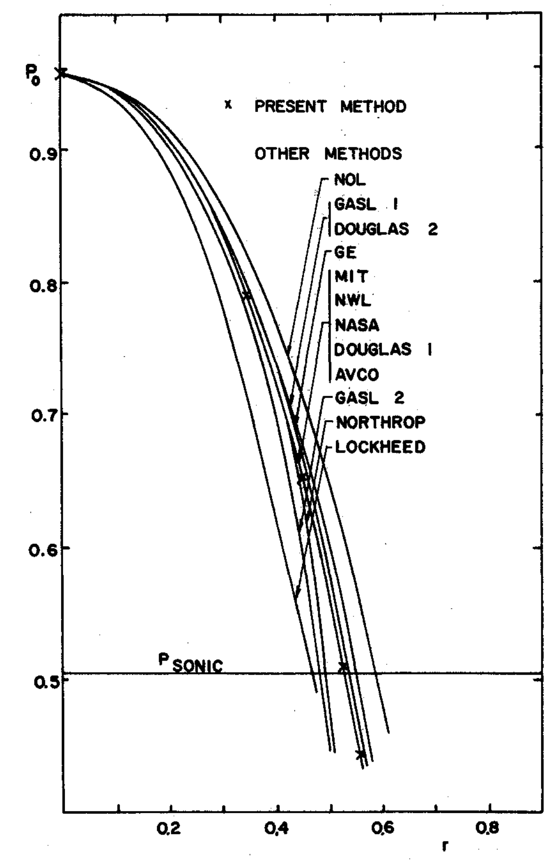


Fig. 6 Steady-state surface pressure (B_b = 9/4, M_∞ = 3, and γ = 1.4).

use Swigart's method of series truncation. Experimental results have been obtained by Pasiuk at Naval Ordnance Laboratory. It can be seen that for the case $B_b = 9/4$ and $M_\infty = 3$, the present solution agrees with the results of five agencies throughout the subsonic region; and for the case $B_b = 4$ and $M_\infty = 3$, the solution agrees with four agencies and nearly matches experiment.

Table 1 tabulates the numerical solution for $M_\infty = 10^4$, $B = 0$, and $\gamma = 1.4$. Also listed are solutions by other methods,[9] including the well refined technique of Lomax and Inouye, the fourth truncation of Van Dyke extrapolated using Shank's transformation, the fifteen-term rational approximation of Moran, and that of Batchelder. All of these solutions are claimed to be accurate up to the fourth significant figure throughout the subsonic region.[9] The present solution for the surface pressure agrees with them in more than four figures at the stagnation point and in more than three figures at the sonic point. But the present solution for the body contour is slightly less accurate at the sonic point.

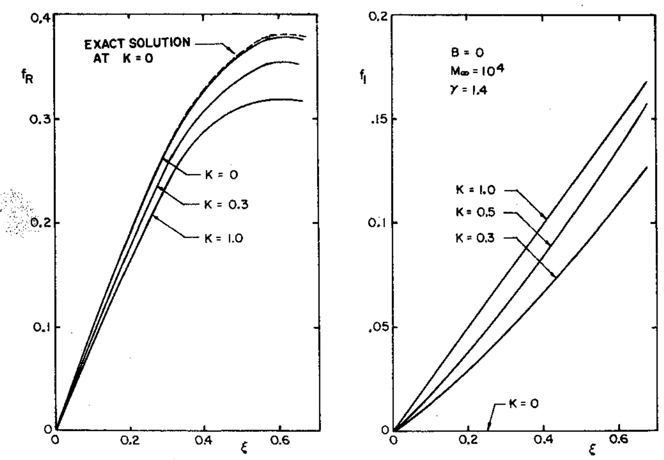


Fig. 7 Shock amplitude in plunging.

Table 1 Numerical solution[a] for $M_\infty = 10^4$, $B = 0$, and $\gamma = 1.4$

	ξ	x	r	p_0
Second truncation	0.0	0.09898	0.0000	0.91969
η_{01} = 0.89557	0.2	0.12157	0.1785	0.86104
η_{02} = −0.07576	0.4	0.18807	0.3541	0.71771
$\bar{p}_{01}$ = 0.73766	0.5	0.23700	0.4405	0.63467
$\bar{p}_{02}$ = −0.44694	0.6	0.29599	0.5258	0.55352
	0.7[b]	0.36484	0.6103	0.47856
	0.8	0.44344	0.6942	0.41199
Third truncation	0.0	0.09898	0.0000	0.91969
η_{01} = 0.89557	0.2	0.12158	0.1785	0.86089
η_{02} = −0.07573	0.4	0.18831	0.3540	0.71751
η_{03} = −0.01510	0.5	0.23749	0.4401	0.63456
$\bar{p}_{01}$ = 0.73766	0.6	0.29683	0.5252	0.55351
$\bar{p}_{01}$ = −0.45100	0.7[b]	0.36612	0.6094	0.47863
$\bar{p}_{03}$ = 0.00140	0.8	0.44519	0.6927	0.41209
Van Dyke's solution		0.09897	0.0000	0.91968
		0.14856	0.2671	0.79596
		0.23606	0.4410	0.63501
		0.36604	0.6094	0.47864
Moran's solution		0.09898	0.0000	0.91969
		0.16444	0.3065	0.76202
		0.26733	0.4875	0.59022
		0.35046	0.5920	0.49377
Batchelder's solution		0.09898	0.0000	0.91969
		0.15559	0.2853	0.78063
		0.25120	0.4641	0.61263
		0.37444	0.6185	0.47078
Lomax and Inouye's solution		0.0989	0.00	0.9198
		0.1535	0.28	0.7853
		0.2354	0.44	0.6361
		0.3948	0.64	0.4528

[a] α = 11.181.
[b] Near the sonic point.

For the case $M_\infty = 10^4$, $B = 0$, and $\gamma = 1.4$, the parameter $\alpha = 11.181$ and two singularities are located at $\xi = \pm 0.925i$ and $\eta = 0.89557$ (or at $x = 0.5r = 0$ and $x = -1.158r = 0$) which are slightly less than Van Dyke's prediction that $\xi = \pm i$. For other cases the locations of the singularities are listed in Table 2. For an infinite Mach number the singularities are located both inside the body and upstream, but for large B and low Mach number the singularities are inside the body only.

4.2 Unsteady Perturbed Solution

The unsteady perturbed solution is obtained by using a direct method of a given body which is found from the steady-

Table 2 Locations of singularity

			Upstream		Inside body	
M_∞	B	γ	x	r	x	r
10^4	0	2.0	−0.88	0.00	0.50	0.00
10^4	0	5/3	−1.04	0.00	0.50	0.00
10^4	0	1.4	−1.16	0.00	0.50	0.00
10^4	0	1.1	−1.51	0.00	0.50	0.00
10^4	0.5	1.4	−3.85	0.00	0.20	0.00
6	0.47	1.4	−4.18	0.00	0.33	0.00
6	1.0	1.4	none		1.0	±0.05
3	2.25[a]	1.4	none		0.65	±0.58
3	4.0[a]	1.4	none		0.44	±0.89

[a] Bluntness for the body surface.

Table 3 Error of the shock function $f(z)$

	$B = 0$		$B = 1$	
	$z = 0$	$z = 0.7^a$	$z = 0$	$z = 0.5^a$
Plunging	−0.019	−0.007	0.001	0.039
Lunging	−0.004	−0.082	−0.006	−0.037

^{a} Near the sonic point.

state solution. The computation is carried out to the second truncation. In the unsteady perturbed solution, the most important function, besides the surface pressure, is the shock wave function $f(z)$. Here the surface pressure has been used for assigning the constants C_1 and C_2 [Eqs. (20)]. Therefore, the function $f(z)$ might be used as a test for the accuracy of the present solution. In the limit of zero frequency, the physical motion of the blunt body corresponds to a coordinate translation, therefore, the function $f(z)$ obtained from the unsteady perturbed solution should approach that from the exact steady-state solution.

Setting $k = 0$ in the unsteady perturbed differential equations for the case $M_\infty = 10^4$ and $\gamma = 1.4$ (Figs. 7 and 8) yields

$$f(z) = 0.981z - 0.9690z^3 + \ldots \quad \text{for } B = 0 \text{ in plunging} \tag{27a}$$

$$f(z) = 1.001z + 0.1553z^3 + \ldots \quad \text{for } B = 1 \text{ in plunging} \tag{27b}$$

$$f(z) = 0.9964 - 1.0790z^2 + \ldots \quad \text{for } B = 0 \text{ in lunging} \tag{27c}$$

$$f(z) = 0.9936 - 0.6068z^2 + \ldots \quad \text{for } B = 1 \text{ in lunging} \tag{27d}$$

Setting $\eta_0 = 1$ in the function $g(\xi)$ of Eqs. (2) and (23) and expanding in powers of ξ gives

$$g(\xi) = \xi - \xi^3 + \ldots \quad \text{for } B = 0 \text{ in plunging} \tag{28a}$$

$$g(\xi) = \xi \quad \text{for } B = 1 \text{ in plunging} \tag{28b}$$

$$g(\xi) = 1 - \xi^2 + \ldots \quad \text{for } B = 0 \text{ in lunging} \tag{28c}$$

$$g(\xi) = 1 - 0.5\xi^2 + \ldots \quad \text{for } B = 1 \text{ in lunging} \tag{28d}$$

Note that at $\eta_0 = 1$, $\xi = z$ and $g(\xi)$ is the exact solution of the function $f(z)$, hence

$$[f(z) - g(\xi)]/g(\xi) = \text{error of the } f(z) \tag{29}$$

Table 3 shows that the error of the shock function $f(z)$ is of the same order of magnitude as that for the perturbed surface pressure, p_1. As mentioned before the error of the p_1 at the high frequency is of the same order as that at zero

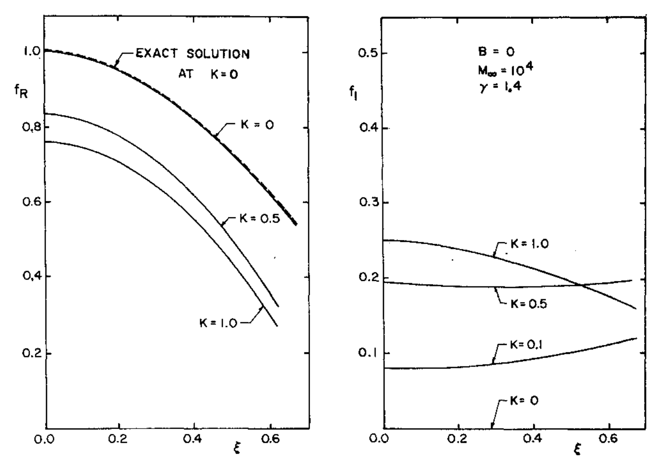


Fig. 8 Shock amplitude in lunging.

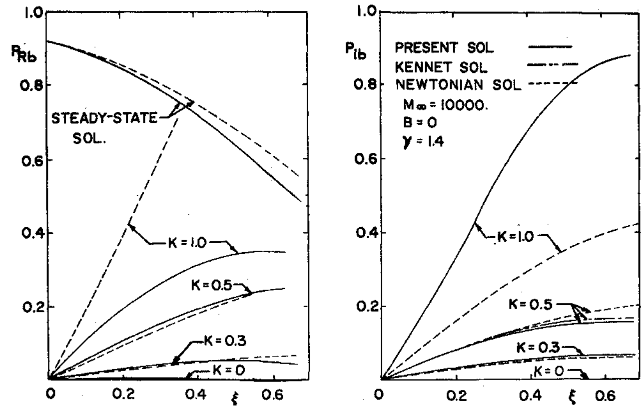


Fig. 9 Unsteady surface pressure parameter in plunging.

and low frequency. Since the $f(z)$ and p_1 are mutually dependent, the same conclusion must be applied for the $f(z)$.

It is well known that the quasi-steady Newtonian solution is valid only at very low frequency.[12] In the rational theory of hypersonic flow, the Newtonian solution must be corrected for the flow variation within the shock layer. For the steady case the flow along the stagnation streamline may be considered as one-dimensional flow. In this case it is concluded that the Newtonian pressure should be multiplied by a factor 0.9097 for $\gamma = 1.4$ (Fig. 9). For the unsteady case such a factor is not easily computed. However, a factor may be obtained by arbitrarily matching a Newtonian solution to the present solution at some particular point and at a particular frequency; for example, the stagnation point for lunging and the neighborhood of the sonic point for plunging at $k = 0.5$. Once the factor is found, all perturbed pressure functions are multiplied by it (Figs. 9 and 10). Using this method Kennet's constant-density solutions are replotted. Both solutions agree in general trend with the present solution, although the constant-density solutions agree better near the sonic point.

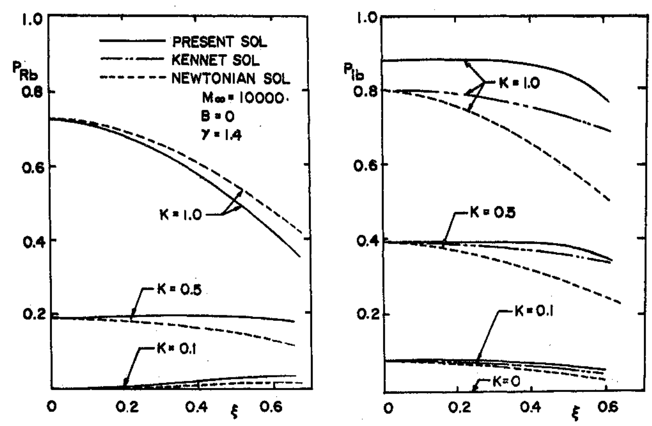


Fig. 10 Unsteady surface pressure parameter in lunging.

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A Turbulent Boundary Layer with Mass Addition, Combustion, and Pressure Gradients

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A subsonic turbulent boundary layer with mass addition and combustion is studied to investigate the effects of combustion on the velocity profiles in constant pressure and accelerating flows. Particular attention is given to determining 1) the extent to which combustion alters the flow and 2) the mechanisms whereby combustion interacts with the flowfield. The experimental results obtained in this study demonstrate that combustion significantly alters the velocity profiles in both constant pressure and accelerating flows. The wall velocity gradients in the combusting flows differ markedly from those of noncombusting flows and show a definite dependence on the pressure gradient. Additionally, the velocity in the flame region of an accelerating flow actually exceeds the freestream value. Analytical results indicate that the experimentally observed changes in the velocity profiles are attributable to the temperature dependence of the local mean density and molecular viscosity. A method of calculating the velocity in a combusting turbulent boundary layer is also presented. Calculated and experimental velocity profiles are compared.

Nomenclature

A = van Driest parameter, $A(\tau_w, \rho_w v_w, dp/dx)$
f' = velocity ratio u/u_e
F = mass injection ratio $\rho_w v_w/\rho v$
p = pressure
u,v = flow velocities
x,y = physical coordinates
δ = boundary-layer thickness
δ_F = flame zone position
ϵ = eddy viscosity
η,ξ = transformed coordinates
θ = momentum thickness
μ = molecular viscosity
ν = kinematic viscosity
ρ = density
τ = shear stress

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Index Category: Boundary Layers and Convective Heat Transfer—Turbulent; Subsonic and Supersonic Airbreathing Propulsion.

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Introduction

COMBUSTING boundary layers are encountered in a number of flow environments of current interest and the need to understand heat, mass, and momentum transfer processes in these environments has stimulated a number of theoretical and experimental studies of this phenomenon. However, review of the literature[1] has indicated that the bulk of these studies have considered laminar boundary layers and only relatively few have investigated turbulent boundary layers.